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Теоретическое исследование перебросовых колебаний наножидкости в связанных каналах при учете зависимости вязкости среды от концентрации частиц

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В работе проведено теоретическое исследование тепловой конвекции наножидкости в подогреваемых снизу связанных каналах. Рассматривались связанные каналы конечной высоты с твердыми высоко теплопроводными границами. Прямое численное моделирование основывалось на уравнениях в частных производных, полученных с помощью галеркинского осреднения по продольной координате. Результирующие уравнения решались методом конечных разностей. По результатам расчетов показано, что нормальная термодиффузия в рассматриваемой конвективной среде оказывает решающее влияние на формирование специфических нелинейных перебросовых колебаний. В ходе численного моделирования были получены поля скорости, температуры и концентрации наночастиц в поперечном сечении на разной высоте каналов. Форма и период этих колебаний в зависимости от надкритичности были проанализированы для разных значений управляющих параметров. Квадратичная зависимость вязкости коллоидного раствора от концентрации наночастиц принималась во внимание в теоретической модели. Показано, что период перебросовых колебаний должен возрастать при уменьшении средней начальной концентрации наночастиц. Нормировка периода колебаний на критическое число Рэлея позволила получить универсальный закон, справедливый для наножидкостей с разными зависимостями вязкости от концентрации. Качественное сравнение показало согласие результатов расчета с известными экспериментальными данными.

Ключевые слова: наножидкости; связанные каналы; тепловая конвекция; термодиффузионное разделение; перебросовые колебания

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A theoretical study of flop-over oscillations in connected channels for nanofluids with concentration dependent viscosity

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Thermal convection of a nanofluid in connected channels heated from below was studied theoretically. Thin channels with boundaries of high thermal conductivity were considered. Direct numerical simulation was based on the set of partial differential equations derived with the help of Galerkin method. The final system was solved by the method of finite differences. It has been shown that normal thermodiffusion exerts principal influence on the formation of particular non-linear flop-over oscillations. Cross-sections of concentration, velocity and temperature were obtained at various heights in the channel. The form and period of these oscillations were analyzed with respect to supercriticality for different values of governing parameters. A quadratic dependence of viscosity on concentration was taken into account. The period of oscillation was found to decrease with the growth of this effect. As a result of the normalization procedure the general law for the period of oscillation has been found. Qualitative agreement between calculation and experiment has been demonstrated.

Keywords: nanofluid; connected channels; thermal convection; thermodiffusive separation, flop-over oscillations

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1. Introduction

For non-uniformly heated fluids, convective flows occur due to the dependence of density on temperature [1]. In the case of binary mixtures, the inhomogeneous concentration distribution also plays a role in convection, and this gives rise to more complex behavior [2]. These effects are dependent on the temperature and concentration gradients. In a mixture, the temperature gradients are responsible for the occurrence of concentration gradients (thermodiffusion). Thermodiffusion can be considered as a cross effect in the general theory of Onsager [3]. The case where a positive temperature gradient creates a negative concentration gradient of the heavier component is known as positive (normal) thermodiffusion.

In vertical connected channels heated from below, positive thermodiffusion and thermoconvection together give rise to observed "flop-over" oscillations in certain intervals of the Rayleigh number, where the velocity of the fluid in the direction of the channel has approximately a square-wave profile, Glukhov and Putin [4]. The calculation of flop-over oscillations was first carried out for binary mixtures in [5], where in accordance with experiment the boundaries were considered to have high thermal conductivity compared to the fluid.

In this paper, we investigate these flop-over oscillations in thin connected channels for nanofluids with heavy particles and viscosity dependent on concentration. Experiments have shown that concentration can have a significant contribution to viscosity for many natural and artificial nanofluids [6, 7]. We suppose that this dependence may be important in providing a more accurate description of flop-over oscillations.

2. Statement of the problem

2.1. Geometry of set-up

Let us consider connected channels (convective loop) oriented vertically and heated from below, as in fig. 1. The channels have finite height h and half-width d, with the condition $h >> d$.

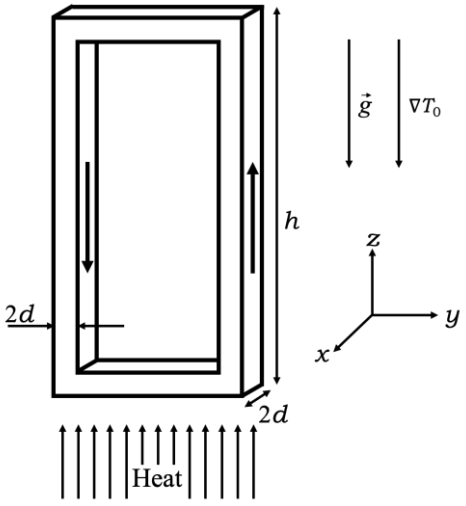


Fig.1. *Geometry of the problem and coordinate system.*

The boundaries are assumed to have high thermal conductivity, in accodance with experiments [4,5]. In the experiments, this was achieved by embedding the channels in a homogeneous metal block. The temperature profile along the centre of the opaque channels was determined using thermocouples. This allowed the flow intensity along the centre of the channel to be estimated quantatively. For small Rayleigh number, it was found that the velocity of the fluid was one dimensional, directed along the channel. At the same time the fields of temperature and concentration were observed to be a function of all spatial coordinates. It was established that the complexity of these fields leads to a highly non-trivial time evolution of this hydrodynamic system.

In our system, the *z*-axis is oriented vertically upwards, the x and y axes lie in the channel's cross section. The temperature difference between the top and bottom of the channels is Θ. Gravity is directed vertically downwards $\vec{g} = (0, 0, -g)$, see fig.1.

2.2. Basic equations

Now let us take the well-known generalized equations of thermal convection in the Boussinesq approximation to describe the behavior of an incompressible fluid in connected channels. This system includes the generalized Navier-Stokes equation, mass conservation law, equations of heat and admixture transfer [1,8]:

$$\frac{\partial \vec{v}}{\partial t} + \ \vec{v}\nabla\ \vec{v} = -\frac{1}{\rho}\nabla p + \nu\Delta\vec{v} + 2\ \nabla\nu\nabla\ \vec{v} + \\ + \nabla\nu\times\ \nabla\times\vec{v}\ + g\ \ \beta_t T - \beta_c C\ \ \vec{\gamma}\,, \quad (2.1)$$

$$\frac{\partial T}{\partial t} + \ \vec{v}\nabla\ T = \chi\Delta T\,, \qquad \mathrm{div}\vec{v} = 0\,, \quad (2.2)$$

$$\frac{\partial C}{\partial t} + \ \vec{v}\nabla\ C = D\ \ \Delta C + \alpha\Delta T\ \ . \quad (2.3)$$

Here, $\vec{v}$, p, T and C are dimensional fields of velocity, pressure, temperature and the concentration of the heavier component. Parameters χ, β_t, β_c are positive coefficients of thermal diffusivity, thermal expansion and dependence of density on concentration, respectively; D and α are the diffusion and thermodiffusion coefficients; ρ is the average density of the fluid; g is the gravitational acceleration; and $\vec{\gamma}$ is the unit vector oriented vertically upward.

The kinematic viscosity, ν, we take to be dependent on concentration quadratically [6], with the form:

$$\nu = \nu_0\ \ 1 + \lambda_d C + \xi_d C^2\ \ , \quad (2.4)$$

where λ_d, ξ_d are positive dimensional parameters dependent on the properties of the nanofluid; ν_0 is the kinematic viscosity of the pure liquid.

The sides of the channels are rigid therefore the velocity vanishes on the boundaries. As a result of the heating from below a linear temperature distribution takes place on the boundaries $T_0 = -z\Theta/h + \Theta$. The unknown fields of velocity $\vec{v}$ and temperature $T = T_0 + T'$ in equations (2.1), (2.2) must satisfy to the following boundary conditions:

$$\vec{v}\Big|_\Gamma = 0\,,$$

$$T'\Big|_\Gamma = 0\,.$$

These account for the existence of non-slip and non-penetration conditions on all sides of the channels, and the high thermal conductivity of the boundaries.

Let us continue the numerical analysis of the equations (2.1)–(2.3) in terms of non-dimensional variables. We shall use following set of units during the simulation: length [Δx, Δy, Δz] – d; time [t] – d^2/ν_0; velocity [v] – ν_0/d; temperature [T] – Θ; concentration [C] – $\beta_t\Theta/\beta_c$; pressure [p] – $\rho\nu_0^2/d^2$. The equations of thermal convection in non-dimensional form can be written as

$$\frac{\partial \vec{v}}{\partial t} + \ \vec{v}\nabla\ \vec{v} = -\nabla p + \nu\Delta\vec{v} + 2\ \nabla\nu\nabla\ \vec{v} + \\ + \nabla\nu\times\ \nabla\times\vec{v}\ + \frac{\mathrm{RaH}}{\mathrm{Pr}}\ T - C\ \ \vec{\gamma}\,, \quad (2.5)$$

$$\frac{\partial T}{\partial t} + \ \vec{v}\nabla\ T = \frac{1}{\mathrm{Pr}}\Delta T\,, \qquad \mathrm{div}\vec{v} = 0\,, \quad (2.6)$$

$$\frac{\partial C}{\partial t} + \ \vec{v}\nabla\ C = \frac{1}{\mathrm{Sc}}\ \Delta C + \varepsilon\Delta T\ \ , \quad (2.7)$$

$$\nu = 1 + \lambda C + \xi C^2\,. \quad (2.8)$$

The following non-dimensional governing parameters are used:

$$\mathrm{Ra} = \frac{g\beta_t\Theta d^3}{\nu_0^2}\,,\ \mathrm{Pr} = \frac{\nu_0}{\chi}\,,\ \mathrm{Sc} = \frac{\nu_0}{D}\,,\ \varepsilon = \frac{\alpha\beta_c}{\beta_t}\,,$$

$$\lambda = \lambda_d\frac{\beta_t\Theta}{\beta_c}\,,\ \xi = \xi_d\left(\frac{\beta_t\Theta}{\beta_c}\right)^2\,,\ H = \frac{h}{d}\,,$$

where Ra, Pr and Sc are the Rayleigh, Prandtl and Schmidt numbers respectively; ε is the non-dimensional thermodiffusion parameter; λ and ξ describe the dependence of viscosity on concentration; H is the non-dimensional height.

In the calculations, we use the straight trajectory approximation for the velocity $\vec{v} = (0, 0, u(x, y, t))$. The form of $u(x, y, t)$ automatically satisfies the incompressibility requirement. As $h >> d$, we limit our considerations to the vertical channels of the convective loop.

The non-dimensional flux of matter is given by

$$\vec{J} = -\nabla\ C + \varepsilon T\ .$$

The non-penetration boundary condition implies

$$J_{n}\Big|_{\Gamma} = 0\,. \tag{2.9}$$

We impose the condition that the net density flux through the two channels is zero:

$$\iint\limits_{s}\ u^{(1)} + u^{(2)}\ \ dxdy = 0\,,$$

where the superscripts denote the left and right channels correspondingly.

3. Mechanical equilibrium

For small Rayleigh number, a mechanical equilibrium state exists, where $\vec{v} = 0$ and $\partial/\partial t = 0$. The non-dimensional temperature distribution at equilibrium is

$$T_{0} = \ -\frac{z}{H} + 1\,.$$

Accounting for the condition on flux and equation (2.7), the concentration field is described by

$$C_{0} = \ \frac{z\varepsilon}{H} + \tilde{C}_{0}\,.$$

The integration constant $\tilde{C}_{0}$ can be expressed in terms of the mean concentration $\bar{C}_{0}$:

$$\tilde{C}_{0} = \ \bar{C}_{0} - \frac{\varepsilon}{2}\,.$$

Let us emphasize the net flux of matter is equal to zero in mechanical equilibrium.

4. Method of solution

In non-equilibrium states, we have 3D fields of concentration, temperature and velocity, which in general depend on time. The full concentration and temperature fields can be written as $C = C_0 + C'$ and $T = T_0 + T'$, where C' and T' are the deviations from mechanical equilibrium. We reformulate the equations (2.5)–(2.7) in terms of these deviations and solve numerically.

To simplify the application of boundary condition (2.9), we introduce the new function $F = C + \varepsilon T$. The associated boundary condition is

$$\nabla F\ _{n}\Big|_{\Gamma} = 0\,.$$

In the following calculations we omit the primes for greater clarity. In the terms of the deviation fields the equation system has the form

$$\frac{\partial u}{\partial t} = -\frac{\partial p}{\partial z} + \nu\Delta_{\perp} u + \\ +\ \nabla\nu\nabla\ u + \frac{\mathrm{RaH}}{\mathrm{Pr}}\ (1+\varepsilon)T - F\ , \tag{4.1}$$

$$\frac{\partial T}{\partial t} + u\frac{\partial T}{\partial z} = \frac{1}{\mathrm{Pr}}\Delta T + \frac{u}{H}\,, \tag{4.2}$$

$$\frac{\partial F}{\partial t} + u\frac{\partial F}{\partial z} = \frac{1}{\mathrm{Sc}}\Delta F + \frac{\varepsilon}{\mathrm{Pr}}\Delta T\,. \tag{4.3}$$

The plane Laplace operator is used in equation (4.1)

$$\Delta_{\perp} = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}.$$

Experimental results [4] indicate the vertical distribution of temperature can be written in terms of two harmonic functions

$$T = T_1(x,y,t)\sin\left(\frac{\pi z}{H}\right) + T_2(x,y,t)\cos\left(\frac{\pi z}{H}\right).$$

Here T_1 and T_2 are amplitudes dependent on the cross section coordinates and time only. Considering the symmetries of equations (4.1)–(4.3) we suggest the following expansion for F:

$$F = F_1(x,y,t) + F_2(x,y,t)\cos\left(\frac{\pi z}{H}\right) + \\ +F_3(x,y,t)\cos\left(\frac{2\pi z}{H}\right).$$

We integrate (4.1) over both channels to remove from this equation the pressure gradient as it does not affect buoyancy forces. For equations (4.2) and (4.3) we apply the Galerkin procedure to find the equations for amplitudes. The resultant system of partial differential equations for the amplitudes u, T_1, T_2, F_1, F_2 and F_3 can be found in the appendix. The equation for u involves only terms anti-symmetric for left and right channels.

The equation system for amplitudes was solved numerically by the method of finite differences in combination with the explicit scheme [9]. One sided and central differences were used to approximate the time and spatial coordinate derivatives respectively, including the convective and diffusive terms [10]. Numerical stability was achieved by choosing an appropriate time step. The initial transitional flows in the results were ignored and stable regimes were investigated.

5. Results and discussion

In the course of direct numerical simulation, we focused our attention on the regime of flop-over oscil-

lations. The characteristic form of these oscillations is illustrated in figure 2.

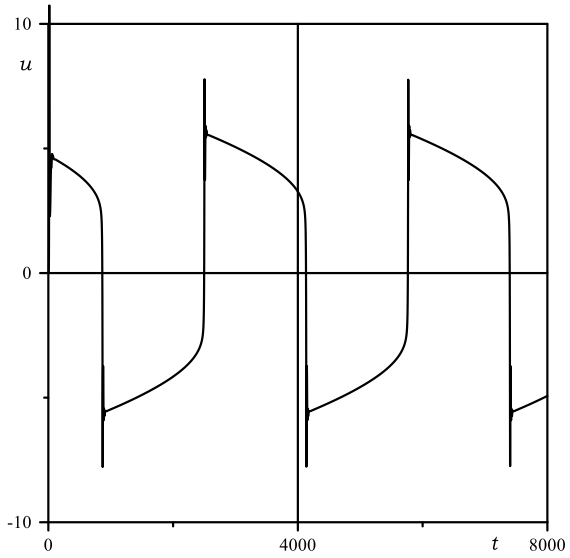


Fig. 2. *The velocity at the center of the channel for* $\lambda = 0.2$, $\xi = 0.5$, *mean concentration* $\bar{C}_0 = 0.1$, Ra = 53.

We observe that an approximately steady flow is established with periodic sharp changes in direction. The mechanism responsible for the observed effects is mainly attributed to the horizontal thermodiffusive separation of the mixture. From experiment [4], the horizontal temperature gradients Θ/d = 3 K/cm are much larger than the vertical gradients Θ/h = 0.3 K/cm, which has a characteristic diffusive separation time of h^2/D ~ 300 days. The horizontal gradients have characteristic separation time d^2/D ~ 3 hours, which coincides in order of magnitude with the oscillation period of the ferrofluid from experiment [4]. Considering first the channel with a flow moving up, the fluid in the center has greater velocity and temperature than the surrounding fluid (fig.3 (*a*)). Therefore, as a result of thermodiffusion, the heavier admixture diffuses towards the slower fluid layer on the boundary. Over time, the channel with upwards flow accumulates the heavier admixture, whilst the opposing channel loses it. When the channel becomes sufficiently heavy, the flow stops abruptly and changes direction.

For our simulation the following values for the governing set of parameters were used: Pr = 10, Sc = 75, ε = 0.3 and H = 30.5. Between simulations the parameters λ, ξ, $\bar{C}_0$ and Ra were varied in order to analyze the effect of concentration dependence of viscosity.

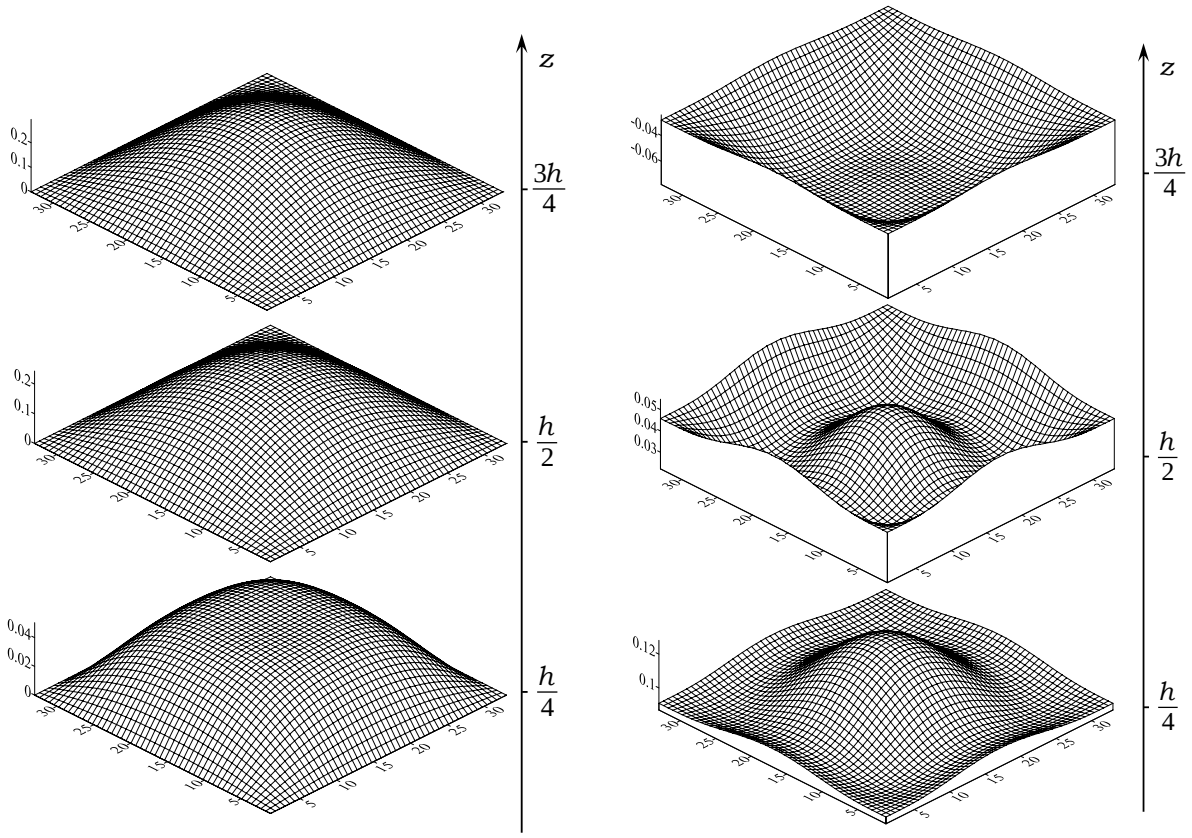
In figure 3 the channel with upward velocity is presented. Figure 3(*a*) shows the temperature deviation profile at various heights along the channel. In our theoretical model we neglect the horizontal parts of the convective loop therefore the full temperature at the ends of the channels must be continuous. At the bottom of the channel the temperature profile has a smaller amplitude than in the upper parts of the channel. This behavior is expected as the inflow at the bottom of the channel has just exited the colder channel.

The concentration deviation field at various heights is illustrated in fig. 3(*b*). For $z = h/4$ and lower the concentration is greater in the center of the channel than near the boundaries. We can see that the concentration of nanoparticles diffuses towards the boundaries as fluid moves up the channel. Towards the top of the channel ($z = 3h/4$) the concentration in the center is smaller. These calculations support the thermodiffusion mechanism outlined earlier in the text and in [5].

The aim of our work is to take into account the effect of concentration dependence of viscosity (2.8). It was not necessary to consider this effect in molecular solutions [5], but experimental data suggest that this may be significant for nanofluids [6].

In our model, the parameters λ and ξ determine the strength of this dependence. The mean concentration $\bar{C}_0$ also affects the convective flows of nanofluids. We are interested in the period of flop-over oscillations for different values of these parameters, as this is accurately measured by experiment [4, 5]. We can obtain from fig. 2 the dimensional value of oscillation period τ: taking the viscosity $\nu = 2.7 \cdot 10^{-6}$ m^2/s and channel width d = 2 mm, we have $\tau \sim 1.6$ hours, which is qualitatively verified by the result from experiment [4]. The quantitative data of period dependence on Rayleigh number from simulation are presented in fig. 4.
It is seen that the period of oscillations increases with Rayleigh number (which is proportional to the temperature difference) approximately linearly. Properties of the nanofluid λ and ξ are considered to be constant. With the decrease of mean concentration the period becomes greater in agreement with experiment [4]. The threshold of flop-over oscillations depends on the mean concentration, λ and ξ. Flop-over oscillations start for each mean concentration at a different critical Rayleigh number Ra_c, which are the lowest values of Ra in fig. 4. Let us attempt to find a universal law for these dependences.

To visualize final data we introduce supercriticality $\mu = (Ra - Ra_c)/Ra_c$. By plotting the period against supercriticality for each mean concentration value we produce a generalized law (fig. 5).



(*a*) (*b*)

Fig. 3. *The fields of temperature (a) and concentration (b) in a cross section of the channel on different heights for* $\lambda = 0.2$, $\xi = 0.5$, *mean concentration* $\bar{C}_0 = 0.1$, Ra = 56.

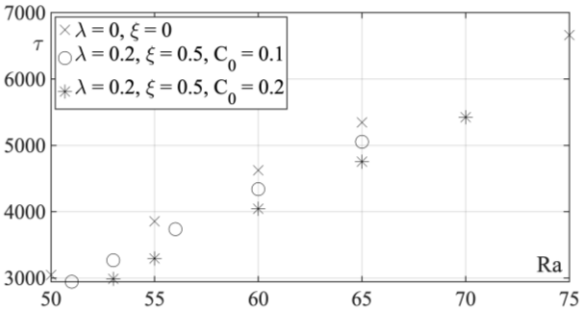


Fig. 4. *Non-dimensional period of oscillations against Rayleigh number for different values of parameters.*

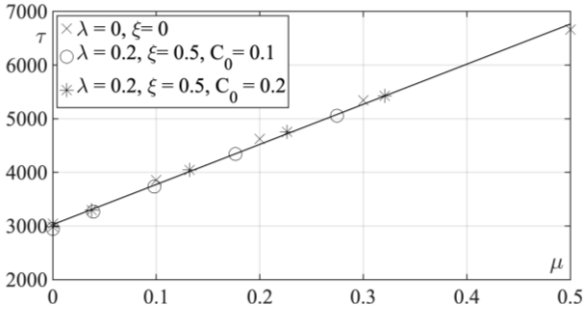


Fig. 5. *Normalized period of oscillations against Rayleigh number for some values of parameters.*

The linear trend line in fig.5 is given by:

$$\tau(\mu) = 7.5\mu + 3.0 \cdot 10^3 .$$

The law suggests that, for a given nanofluid, there is a minimum period for flop-over oscillations regardless of mean concentration. For our parameters, this is ~ $3{\cdot}10^3$. The period of flop- over oscillation in a given nanofluid (constant Sc, Pr, and ε) for small values of supercriticality can to a good approximation be described as having a linear dependence on Θ, a convenient parameter to measure in experiments. The flop-over regime's dependence on concentration must be contained in the critical value of temperature $\Theta_c = \Theta_c(\lambda, \xi, \bar{C}_0, \ldots)$. Our numerical results so far suggest Θ_c increases with $\bar{C}_0$. Many further simulations would have to be performed to determine a more explicit form of Θ_c; however its value is easily attainable from experiment.

Though the model provided by this paper allows for a description of flop-over oscillations and dependence on Rayleigh number and mean concentration it is by no means complete. In this simulation we increased the Rayleigh number (experimentally realized by increasing the temperature difference) without changing

other parameters dependent on temperature difference (e.g. λ, ξ, $\overline{C}_0$). Further research may wish to take into account the temperature dependence of the other governing parameters. Possible future work could even include varying the Schmidt number with potential concentration dependence.

6. Conclusion

Non-steady thermal convection of a nanofluid in connected channels with boundaries of high heat conductivity was investigated theoretically guided by experiment. In our model we assume the liquid moves predominantly vertically, adjacent to heat-conducting boundaries. We suggest that the separation of components in the horizontal plane by thermodiffusion is responsible for the complex "flop-over" oscillatory regime. Our direct numerical simulation based on hydrodynamic equations for the convective loop is in qualitative agreement with the results of experiments [4, 5]. The simulations in this paper have demonstrated that the introduction of concentration dependent viscosity clearly affects the period of flop-over oscillations in a convective loop. This brings numeric simulations qualitatively in line with the results of experiments on nanofluids.

We are grateful for the financial support of the scholarship provided by British Petroleum. We thank the University of Oxford's Career Service and the International Department at Perm State University for providing the research opportunity.

Appendix

The following final partial differential equation system gives the time evolution for amplitudes $T_1(x, y, t)$, $T_2(x, y, t)$, $u(x, y, t)$, $F_1(x, y, t)$, $F_2(x, y, t)$, $F_3(x, y, t)$. After solving for these amplitudes, we return to the full values of temperature, velocity and concentration.

$$\frac{\partial u}{\partial t} = \left(1 + \lambda\overline{C_0} + \xi\overline{C_0}^2 + \frac{\varepsilon^2\xi}{12}\right)\Delta_\perp u + \frac{2H\mathrm{Ra}}{\mathrm{Pr}\,\pi}\left[1 + \varepsilon\ T_1 - \frac{\pi}{2}F_1\right] +$$
$$+ \xi\left\{\left[\left(\frac{1}{2}\right)\ 2F_1^2 + F_2^2 + F_3^2 - 2\varepsilon F_2T_2 + \varepsilon^2\ T_1^2 + T_2^2 \quad - \left(\frac{4\varepsilon}{3\pi}\right)\ 3F_1 + F_3\ T_1\right]\Delta_\perp u +\right.$$
$$\left. + \left[\ 2F_1\nabla F_1 + F_2\nabla F_2 + F_3\nabla F_3 \ - \varepsilon\ T_2\nabla F_2 + F_2\nabla T_2 \ + \varepsilon^2\ T_1\nabla T_1 + T_2\nabla T_2 \ - \right.\right.$$
$$\left.\left. - \left(\frac{4\varepsilon}{3\pi}\right)\ 3\ T_1\nabla F_1 + F_1\nabla T_1 \ - \ T_1\nabla F_3 + F_3\nabla T_1 \ \right]\nabla_\perp u\right\},$$

$$\frac{\partial T_1}{\partial t} - \frac{u\pi}{H}T_2 = \frac{1}{\mathrm{Pr}}\left[\Delta_\perp T_1 - \left(\frac{\pi}{H}\right)^2 T_1\right] + \frac{4u}{\pi H}, \quad \frac{\partial T_2}{\partial t} + \frac{u\pi}{H}T_1 = \frac{1}{\mathrm{Pr}}\left[\Delta_\perp T_2 - \left(\frac{\pi}{H}\right)^2 T_2\right],$$

$$\frac{\partial F_1}{\partial t} - \frac{2u}{H}F_2 = \frac{1}{\mathrm{Sc}}\Delta_\perp F_1 + \frac{3\varepsilon}{\pi\,\mathrm{Pr}}\left[\Delta_\perp T_1 - \left(\frac{\pi}{H}\right)^2 T_1\right], \quad \frac{\partial F_2}{\partial t} - \frac{16u}{3H}F_3 = \frac{\varepsilon}{\mathrm{Pr}}\left[\Delta_\perp T_2 - \left(\frac{\pi}{H}\right)^2 T_2\right] - \frac{1}{\mathrm{Sc}}\left(\frac{\pi}{H}\right)^2 F_2,$$

$$\frac{\partial F_3}{\partial t} + \frac{4u}{3H}F_2 = \frac{1}{\mathrm{Sc}}\left[\Delta_\perp F_3 - 4\left(\frac{\pi}{H}\right)^2 F_3\right] + \frac{4\varepsilon}{3\pi\,\mathrm{Pr}}\left[\Delta_\perp T_1 - \left(\frac{\pi}{H}\right)^2 T_1\right], \quad \nabla_\perp = \frac{\partial}{\partial x}\vec{e}_x + \frac{\partial}{\partial y}\vec{e}_y.$$

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