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ANALYSIS OF PARTY SYSTEMS BY MEASURES OF CONCENTRATION OF INEQUALITY AND ASYMMETRY OF THE PARETO CURVE

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Abstract
The article proposes to assess party systems using the Hoover Index (HI) and the Pareto curve skewness coefficient, PAC. Conceptually, HI is one of the simplest and most intuitive measures of inequality concentration, designed to determine the proportion of votes that must be redistributed from parties that received at least the average number of votes in elections to other parties in order to achieve an even distribution of votes. The PAC determines which parties contribute the most to overall party inequality as measured by HI. For the typology of inequality concentration in party systems, the generalized Pareto principle is used. When applying the new concept to the analysis of party systems in 18 European countries (158 electoral cases), it was found that most of them had a left-wing skewness of the Pareto curve and a concentration of inequality close to the proportion of the Pareto principle. The proposed method for assessing party systems can be considered as an independent tool, or as an addition to the currently widely used Laakso-Taagepera effective numbers of parties.

Keywords: party system; effective number of parties; Pareto curve; asymmetry; concentration of party systems; Pareto principle; concentration typology.

Currently, the most widely used measure for the fragmentation of party systems based on election results is the “effective number of parties, ENP” introduced by Laakso and Taagepera. The conceptual idea of the ENP and its varieties (Dunleavy & Boucek, 2016; Gaines & Taagepera, 2014; Golosov, 2010; Molinar, 1991; Rae, 1967; Taagepera, 1999) is to combine the number and size distribution of parties into a single fragmentation factor equal to the number of "important parties." The advantage of the ENP is the simplicity of calculations, while the disadvantage is the ambiguity in measuring party systems, since different scenarios for the distribution of votes in elections can correspond to the same value of the ENP. Also, being an artificial measure of party systems, the ENP has no intuitive meaning, which makes it more complicated to interpret the results obtained (Bogaards, 2004; Golosov, 2010; Dunleavy and Boucek, 2016; Суманеев, 2017; Magyar, 2022). In this regard, it is of interest to supplement the ENP with other parameters to reduce the ambiguity in measuring the fragmentation of party systems. Hereby we propose to use the Hoover index, HI, and the Pareto curve asymmetry coefficient, PAC, as additional parameters for the fragmentation of party systems. Conceptually, the HI is the simplest and most intuitive measure of the concentration of inequality, which determines the proportion of votes that must be redistributed to achieve equality of parties. The PAC determines which parties contribute the most to the overall inequality as measured with the HI.

Methods of Measurement

Hoover index. Let the number of parties that received at least one vote in the elections be n. We denote the proportion of votes (party size) ranked in descending order by w_r. Then, we denote the cumulative sum of proportion values for votes$l$ of parties as $S_l = \sum_{r=1}^{l} w_r$. By convention, $S_n = 1$.

The ENP is a real number calculated using the formula (Laakso and Taagepera, 1979):

$$\mathrm{ENP} = \frac{1}{\sum_{r=1}^{n} w_r^2}. \quad (1)$$

Successful parties will mean parties that won a proportion of votes of not less than the average value of $\bar{w} = 1/n$ in the elections. Let the number of successful parties (Succ NP) be equal to the natural number

m. Therefore, the proportion of successful parties $p_m = m/n$, and the number of votes gained by successful parties is equal to S_m.

The Hoover index can be mathematically defined as (Hoover, 1936):

$$\mathrm{HI} = S_m - p_m, \quad (2)$$

It can be seen from (2) that HI is the portion of votes that would have to be redistributed, i.e., taken from successful parties to other parties to have equal distribution of votes. This is the reason the HI is often referred to as the Robin Hood index. The HI is also known as the Pietra index (Pietra, 1915) or the Schutz index (Schutz, 1951).

The Hoover index satisfies the inequality $0 \leq \mathrm{HI} < 1$. The lower limit of the HI is obtained when there is a complete equality of votes, while the upper limit is reached when all voters voted for one party. On a Pareto chart, the HI is equivalent to the longest vertical distance between the Pareto curve and the 45-degree line representing perfect equality.

Pareto curve. The piecewise linear graph $S_t = S(p_t)$ is referred to as the Pareto curve, PC. PCs can be symmetrical or asymmetrical with respect to the alternate diagonal drawn from (0, 1) to (1, 0) of the unit square. PC symmetry means that the curve to the left of the alternative diagonal is a mirror image of the portion of the curve to the right of that diagonal. The PC symmetry condition is mathematically defined by the following equation (Kakwani, 1980):

$$p_m + S_m = 1. \quad (3)$$

When substituting (3) into (2), we find that the proportion of successful parties for symmetrical PCs is equal to

$$p_m = 0.5(1 - \mathrm{HI}). \quad (4)$$

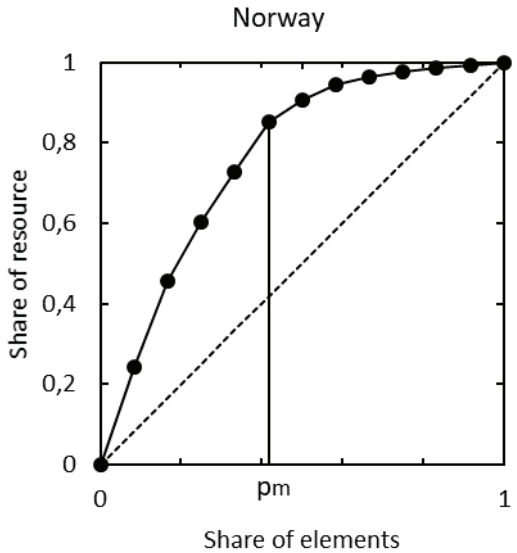
Asymmetric PCs are skewed up or down. For up-skewed PCs, the "longer part" is to the left of p_m (the left-hand asymmetry), while for down-skewed PCs, the "longer part" is to the right of p_m (the right-hand asymmetry). To measure the PC skewness, we will calculate the asymmetry coefficient (PAC) using the formula

$$\mathrm{PAC} = 1 - p_m - S_m. \quad (5)$$

It can be seen from relation (5) that for PCs with right-hand asymmetry $\mathrm{PAC} > 0$, and for PCs with left-hand asymmetry, $\mathrm{PAC} < 0$.

PAC describes an important aspect of the Pareto curve shape. It shows which parties contribute the most to the overall inequality of parties as measured with the HI. If $\mathrm{PAC} < 0$, then the inequality is primarily due to the relatively large number of small parties. If $\mathrm{PAC} > 0$, then the inequality is primarily associated with the few largest parties. Note that the asymmetry coefficient (5) describes the asymmetry $S(p)$ only in the neighborhood of p_m.

As an example, dots in Figure 1 show the PCs of voting results of the parliamentary elections in Norway in 2001 (a) and Bulgaria in 1994. (b). As we can see, the PC of voting results of the parliamentary elections in Norway has a left-hand asymmetry, while in Bulgaria it has a right-hand asymmetry.



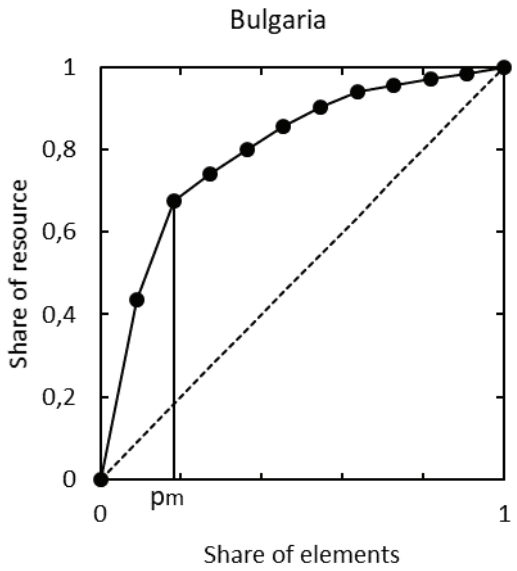


Figure 1. Pareto curves of voting results in Norway in 2001 and Bulgaria in 1994

Typology of parties. In the late 19th century, Vilfredo Pareto published his research findings on wealth inequality in Italy in the form of a 20/80 ratio — 20% of families owned approximately 80% of all land (Pareto and Page, 1971). In the mid-20th century, Joseph M. Juran, after reading the work of Vilfredo Pareto, came to the conclusion that the 20/80 proportion adequately describes the principle of quality management that he had previously discovered, i.e. "the vital few and the trivial many", and later renamed his quality principle as Pareto principle (Juran, 1954, 1975). In 1996, Epstein and Axtell, using an agent-based model called SugarScape, showed that the 20/80 ratio is a natural phenomenon (Epstein and Axtell, 1996).

The principle of concentration of inequality discovered by Vilfredo Pareto gave rise to numerous studies of similar patterns in systems of different nature. The findings showed that in social systems, approximately 20-30% of a resource utilized provide 70-80% of results associated with this resource; accordingly, the remaining 80-70% provide only 30-20% of results (Zipf, 1949). The boundaries of the Pareto principle expanded by George Kingsley Zipf have embraced fundings on marketing systems (McCarthy and Winer, 2019; Sharp, Romaniuk and Graham, 2019), economic systems (Grachev, 2022), and party systems (Грачев, 2011, 2012).

From Zipf's development of the Pareto principle, it follows that most common HI values should be in the range from 0.4 to 0.6. Based on this, we can classify the concentration of inequality into three types:

- Type I — high concentration, $0.6 < HI$,
- Type II — moderate concentration, $0.4 \leq HI \leq 0.6$,
- Type III — low concentration, $HI < 0.4$.

Applying New Measures to European Countries

The official electoral commissions' websites of were used as empirical data sources. 18 European countries were selected to measure the state of party systems. A total of 158 election cases was processed. The preparation of empirical data for further analysis of party systems included selecting parties that received at least one vote in parliamentary elections and normalizing the sum of proportion values of all parties to 1. Figure 2 shows a scatterplot of Share of Succ NP and HI, and Figure 3 shows a scatterplot of the ratio of the number of successful parties to the number of effective parties and HI.

It can be seen from the scatterplot on Figure 2 that the regression line describing the relationship between the proportion of successful parties and the Hoover index matches equation (4) that describes the theoretical dependence of the proportion of successful parties on the Hoover index in systems with symmetrical PCs. The scatterplot on Figure 3 shows that for low and moderate concentration party systems, the number of successful parties is less than the effective number of parties as introduced by Laakso and Taagepera. And conversely, in high concentration party systems, Succ NP values are higher than ENP values.

The behavior of HI and PAC of party systems in 18 European countries is shown in Figure 4.

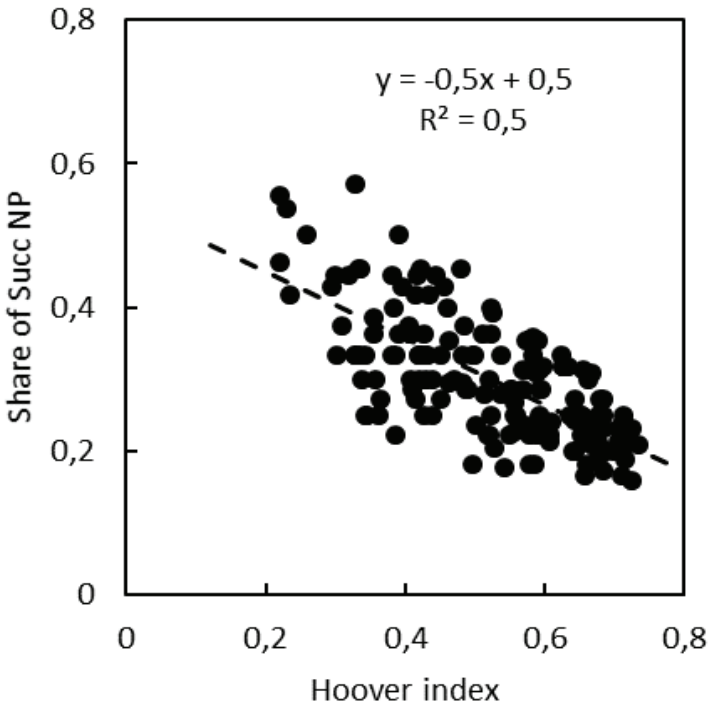


Figure 2. Scatterplot of Succ NP and HI

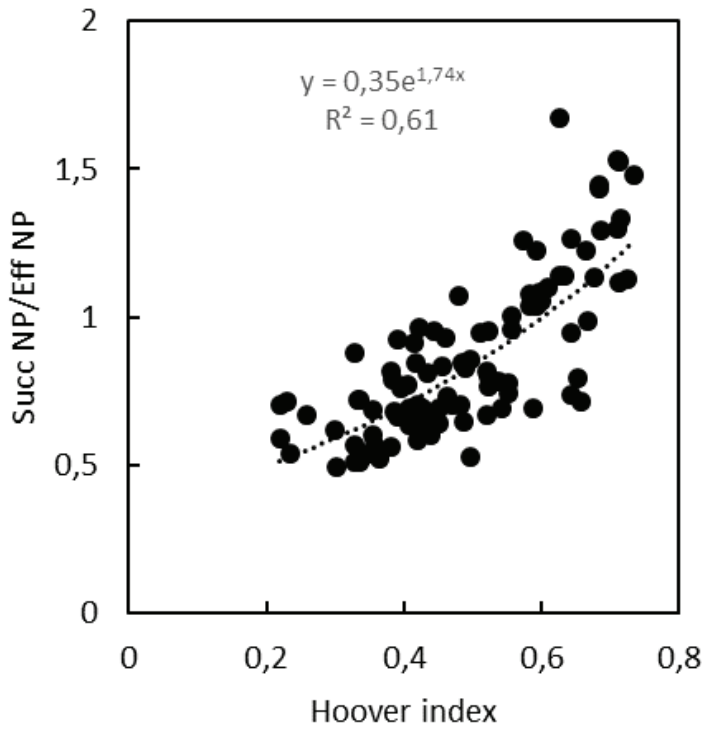
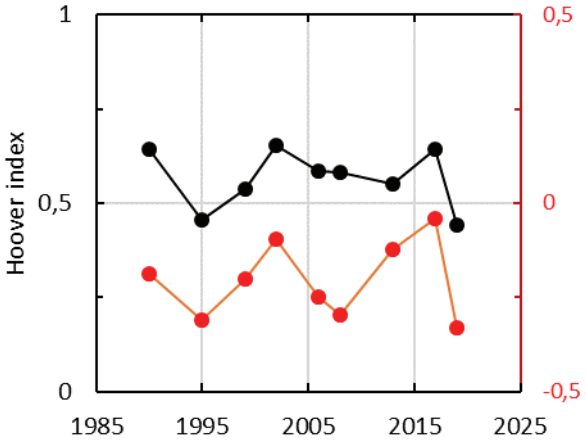


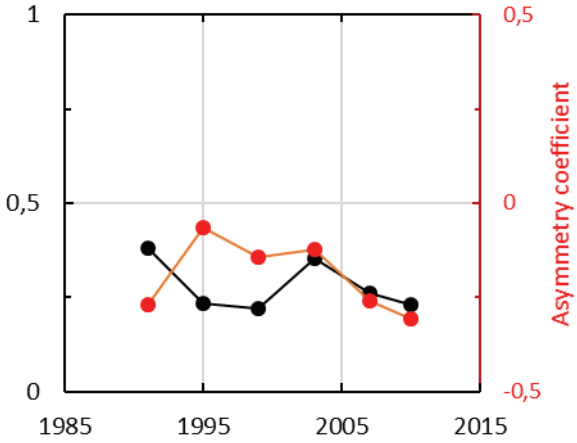
Figure 3. Scatterplot of the ratio of the Succ NP to the ENP and HI

The charts in Figure 4 show that each of the party systems under examination has come its path of development. Thus, the spike in the left-hand asymmetry of the PC of the party system in Germany occurred after the reunification of the FRG and the GDR took place. There are also general patterns. For example, right-hand PC asymmetry was seen in 10 countries — Bulgaria, Denmark, France, Iceland, Norway, Portugal, Russia, Spain, Sweden, and Switzerland. Only left-hand PC asymmetry occurred in Austria, Belgium, Finland, Germany, Italia, Netherlands, Slovakia, Switzerland, and United Kingdom. The rapid decline in left-hand PC asymmetry in the UK and France began about the same time.

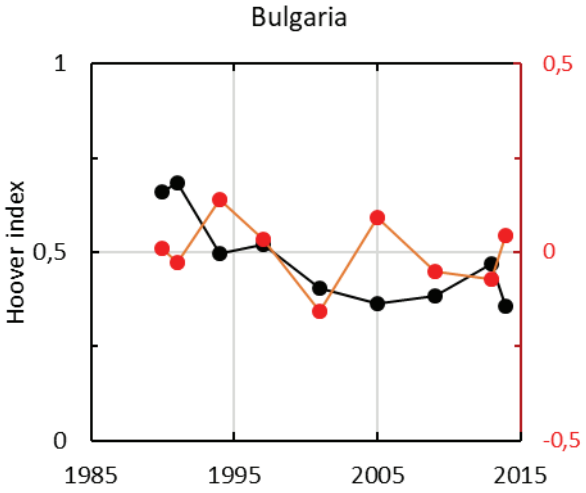
(a)


Austria
Hoover index
1
0,5
0
0,5
0
-0,5
1985
1995
2005
2015
2025

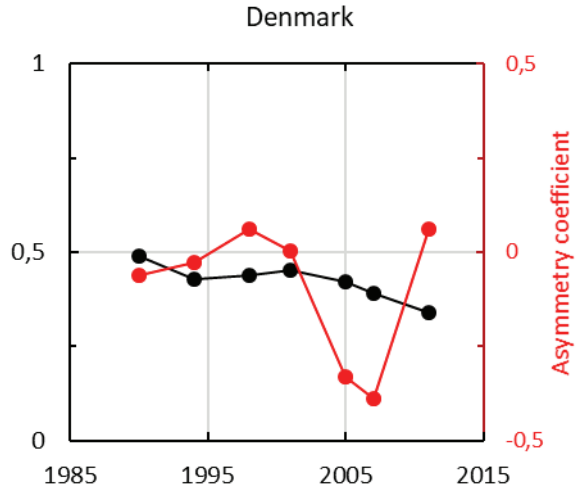
(b)


Belgium
1
0,5
0
0,5
0
-0,5
Asymmetry coefficient
1985
1995
2005
2015

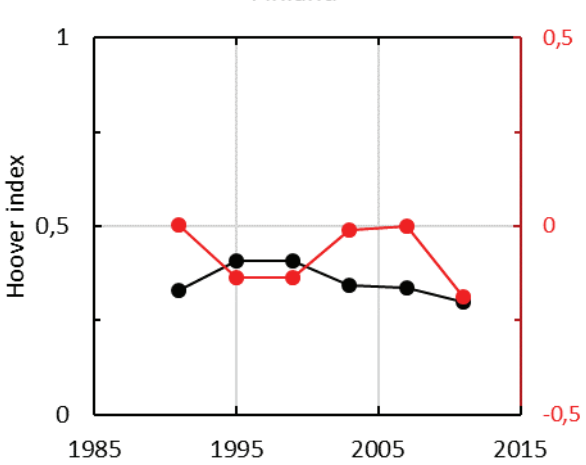
(c)


Bulgaria
Hoover index
1
0,5
0
0,5
0
-0,5
1985
1995
2005
2015

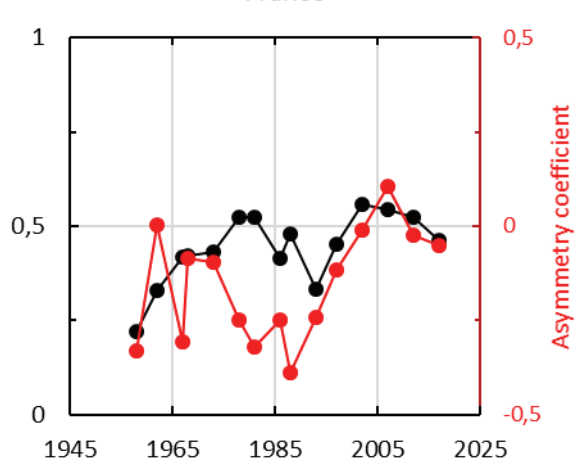
(d)


Denmark
1
0,5
0
0,5
0
-0,5
Asymmetry coefficient
1985
1995
2005
2015

(e)

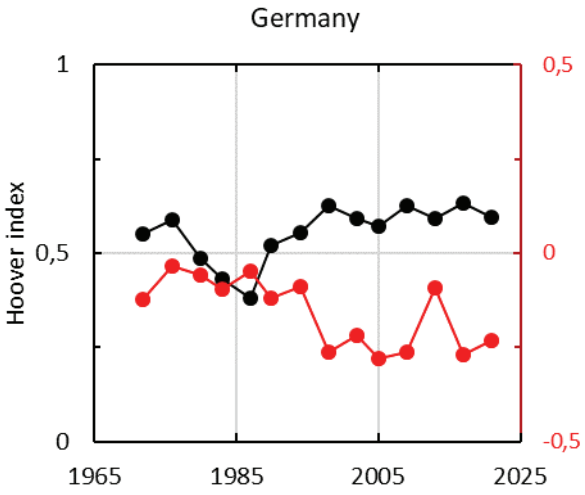

Finland
Hoover index
1
0,5
0
0,5
0
-0,5
1985
1995
2005
2015

(f)

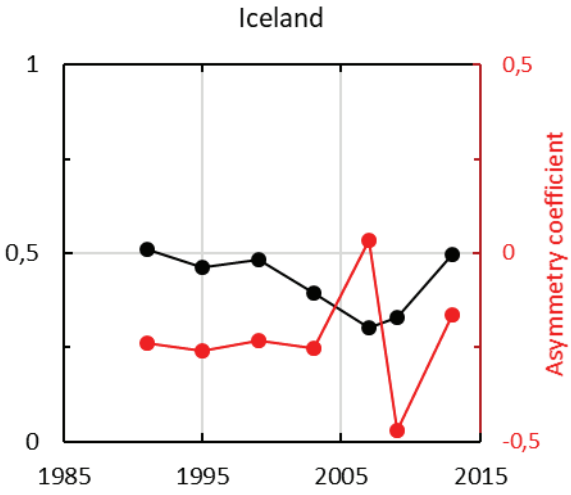

France
1
0,5
0
0,5
0
-0,5
Asymmetry coefficient
1945
1965
1985
2005
2025

(g)

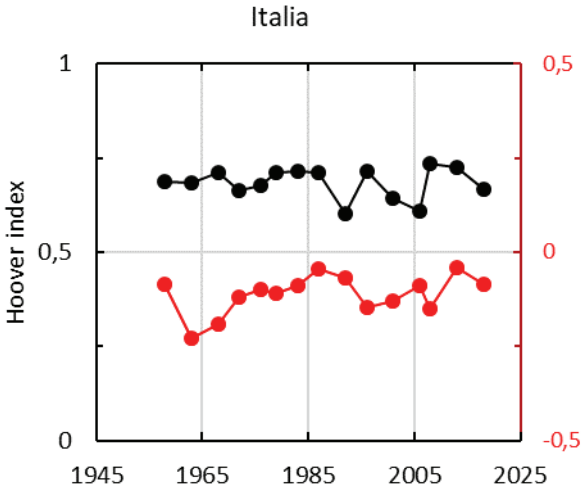
(h)


Germany
Hoover index
Asymmetry coefficient
1
0,5
0
0,5
0
-0,5
1965
1985
2005
2025

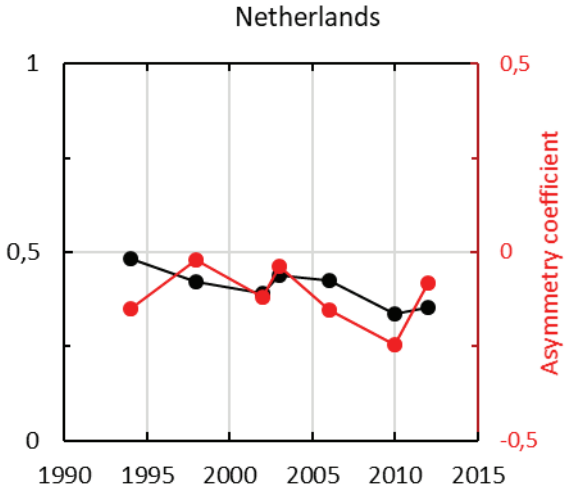
(i)


Iceland
Asymmetry coefficient
1
0,5
0
0,5
0
-0,5
1985
1995
2005
2015

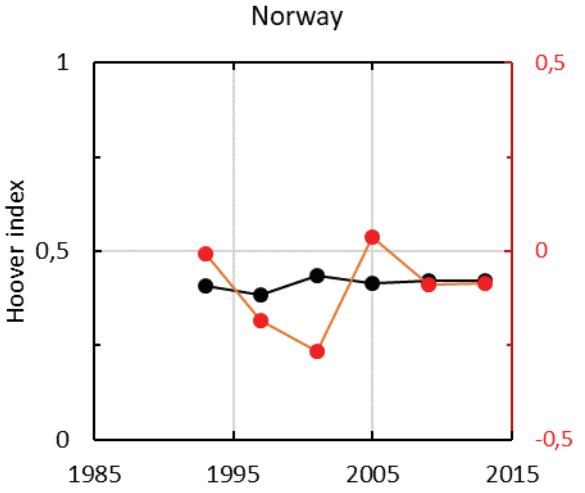
(j)


Italia
Hoover index
Asymmetry coefficient
1
0,5
0
0,5
0
-0,5
1945
1965
1985
2005
2025

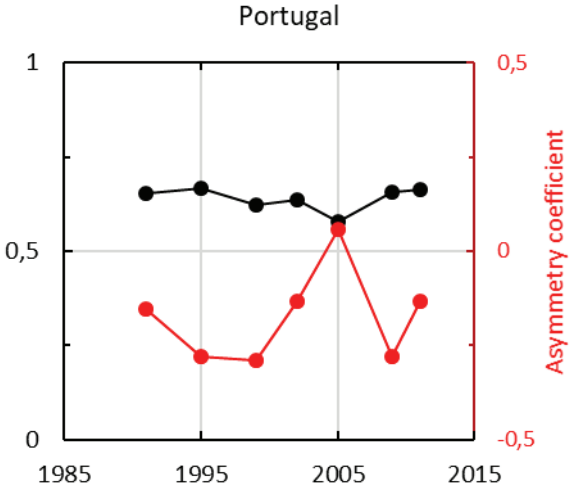
(k)


Netherlands
Asymmetry coefficient
1
0,5
0
0,5
0
-0,5
1990
1995
2000
2005
2010
2015

(l)


Norway
Hoover index
Asymmetry coefficient
1
0,5
0
0,5
0
-0,5
1985
1995
2005
2015

(m)


Portugal
Asymmetry coefficient
1
0,5
0
0,5
0
-0,5
1985
1995
2005
2015

(n)

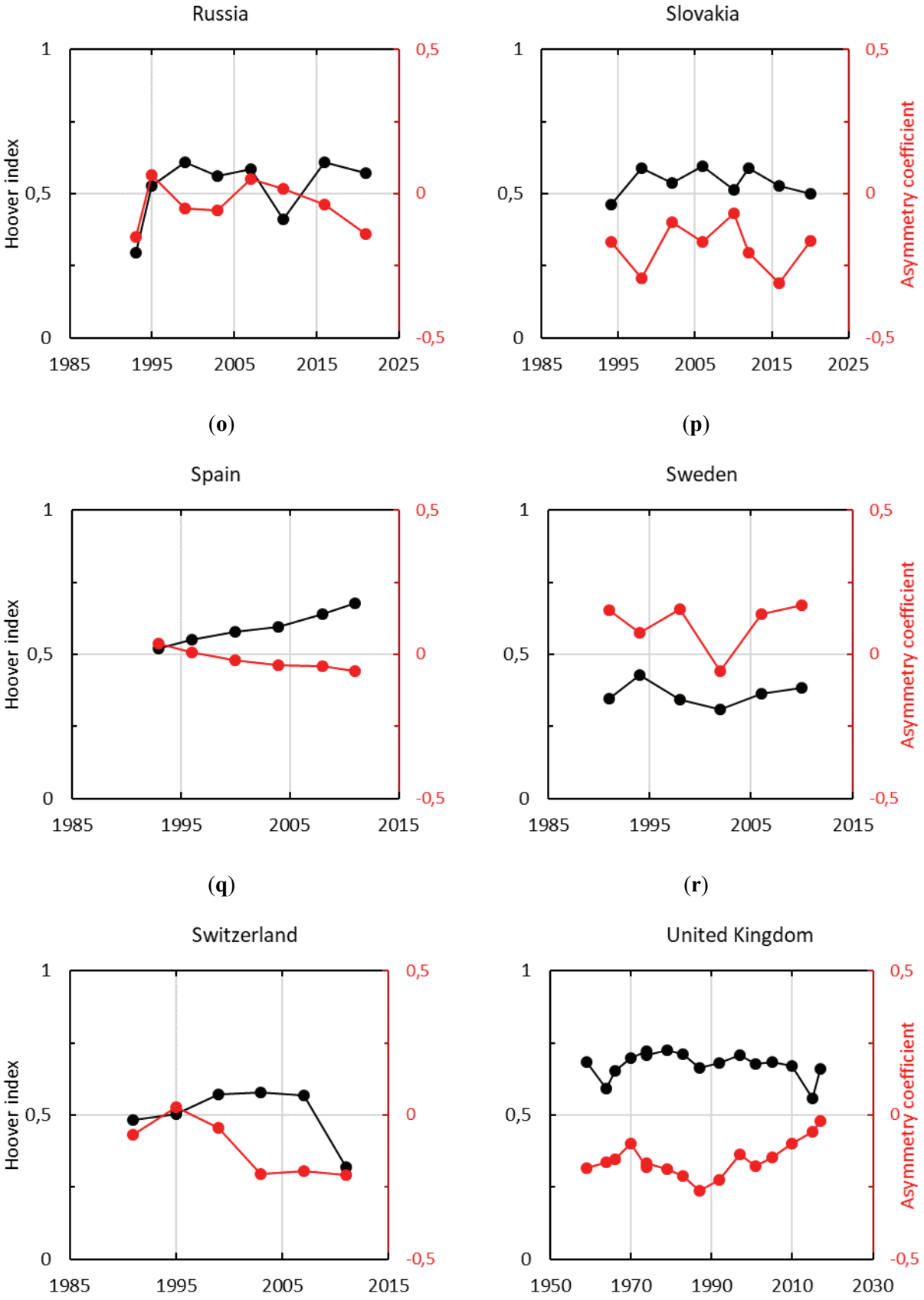


Figure 4. The behavior of state of the party systems over time

The results of identification of party systems by concentration of inequality are presented in Table. It can be seen that in these years, high concentration of inequality was in 4 countries, while moderate and low

concentration were spread evenly in the rest of the countries. PC had left-hand asymmetry in 16 countries and right-hand asymmetry only in Bulgaria and Sweden.

Table

Results of identification of party systems in 18 European countries by concentration of inequality in fixed years of the 21st century

Country	Year	Concentration of inequality, HI			PAC
		High	Moderate	Low	
Austria	2019		0.44		-0.33
Belgium	2010			0.23	-0.31
Bulgaria	2014			0.36	0.04
Denmark	2011			0.34	0.06
Finland	2011			0.3	-0.19
France	2017		0.46		-0.05
Germany	2021	0.6			-0.23
Iceland	2013		0.50		-0.16
Italy	2018	0.67			-0.08
Netherlands	2012			0.35	-0.08
Norway	2013		0.42		-0.09
Portugal	2011	0.66			-0.13
Russia	2021		0.57		-0.14
Slovakia	2020		0.50		-0.17
Spain	2011		0.56		-0.06
Sweden	2010			0.38	0.17
Switzerland	2011			0.32	-0.21
United Kingdom	2017	0.66			-0.02

* * *

In this work, the Hoover index and asymmetry coefficient of the Pareto curve were tested to address the measuring party systems. When applying the new concept to analyze party systems in 18 European countries (158 election cases), we found that most of them had a concentration of inequality close to the Pareto principle and a left-hand asymmetry PC, which distinguishes significantly the PC of party systems from the PC of income and urban settlement systems, which featured both right-hand and left-hand asymmetry (Clementi et al., 2019; Grachev, 2022).

The dominance of left-hand asymmetry in the PC of party systems can be explained by the struggle of political parties for votes of electors, while most of them are at the center of the ideological model (Downs, 1957). The cause for decline in the left-hand PC asymmetry which began in the UK in 1987 and France in 1988 may have been voters who sought alternatives that would better agree with their political preferences (Spoon and Klüver, 2019).

A slight asymmetry in 9 out of 18 countries seems to suggest that at the beginning of the 21st century, those countries had similar conditions for the functioning of party systems. This con-clusion follows from the universal principle of symmetry formulated by Pierre Curie (1894): the function-ing of similar systems of any nature is only possible in symmetric media, while the functioning of systems with individual properties is only possible in asymmetric media.

It is known that a decrease in the concentration of inequality in party systems will result in increased cooperation between parties and a larger number of possible coalition alternatives (Vayrynen, 1972). As a similar effect occurs with an increase in left-hand PC asymmetry, one of the further research objectives should be to look at the influence of PC asymmetry on the number of coalition alternatives, followed by the development of a typology of systems based on PC asymmetry.

Consequently, the use of the Hoover index and the Pareto curve asymmetry coefficient provides new opportunities for understanding the development of party systems. The methodology for analyzing the systems proposed in this work can be considered as an independent tool, or as an addition to the meth-od for measuring party systems.

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АНАЛИЗ ПАРТИЙНЫХ СИСТЕМ ПО ПОКАЗАТЕЛЯМ КОНЦЕНТРАЦИИ НЕРАВЕНСТВА И АСИММЕТРИИ КРИВОЙ ПАРЕТО

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Аннотация
Предлагается оценить состояние партийных систем с помощью индекса Гувера (HI) и коэффициента асимметрии кривой Парето (РАС). Концептуально HI является одним из самых простых и интуитивно понятных мер концентрации неравенства, предназначенной для определения доли голосов, которую необходимо перераспределить от партий, получивших на выборах не менее среднего числа голосов, к другим партиям для достижения равномерного распределения голосов. РАС определяет, какие партии вносят наибольший вклад в общее партийное неравенство, измеряемое HI. Для типологии систем по концентрации неравенства используется обобщенный принцип Парето. Применив новую концепцию к анализу партийных систем в 18 европейских странах (158 электоральных случаев), было установлено, что большинство из них имеют левосторонний перекос кривой Парето и концентрацию неравенства, близкую к пропорции принципа Парето. Предложенный метод оценки состояния систем может рассматриваться как самостоятельный инструмент или как дополнение к широко используемому в настоящее время эффективному числу партий Лаакса-Тагапера.

Ключевые слова: партийная система; эффективное число партий; кривая Парето; асимметрия; концентрация партийных систем; принцип Парето; типология концентрации.